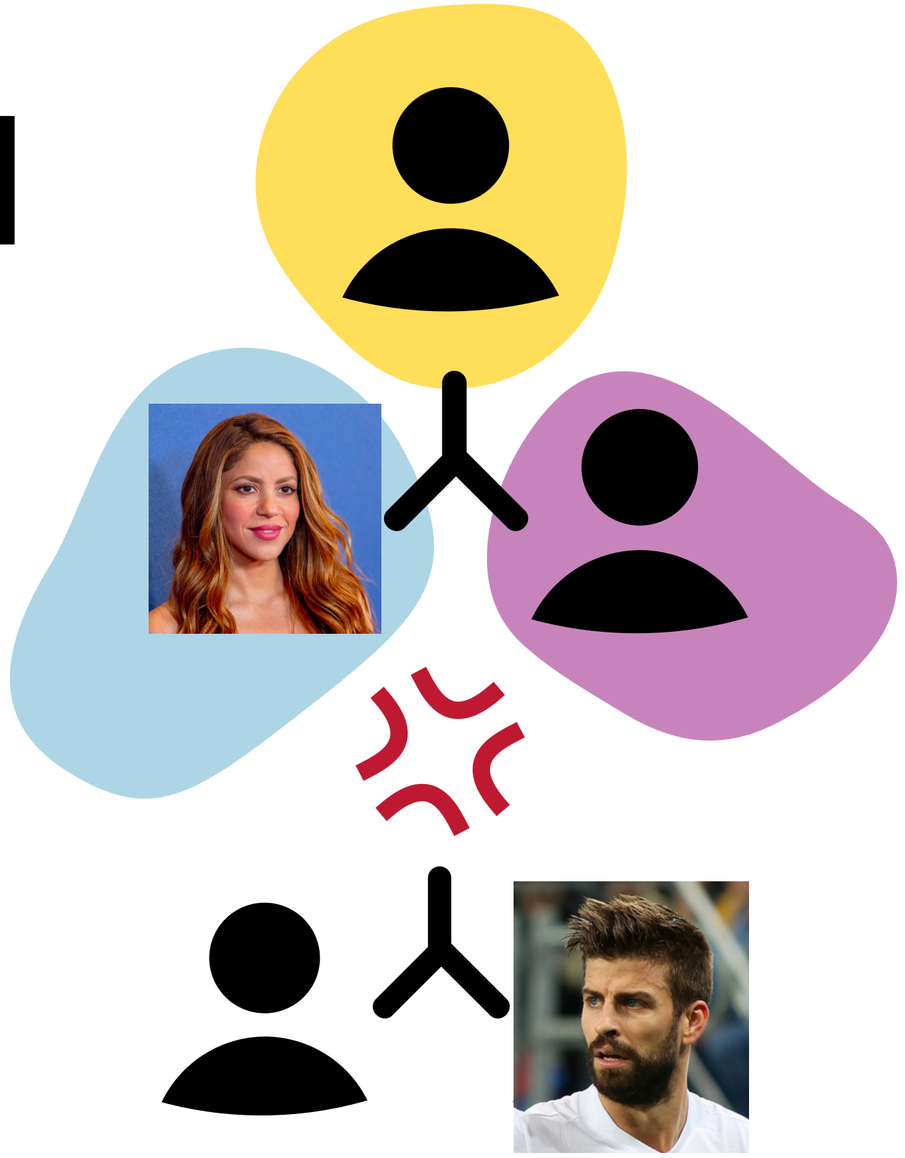
Friendship Signed Networks

Game theoretical ABM model and data-driven characterization

Miguel Ángel González Casado,
Emma Fraxanet, Fernando Diaz-Diaz,
Elena Candellone, Irene Ferri
Satellite collaborators: Samuel, Shahriar and Lucas



Outline

Empirical dataset

Network of friendship in high school

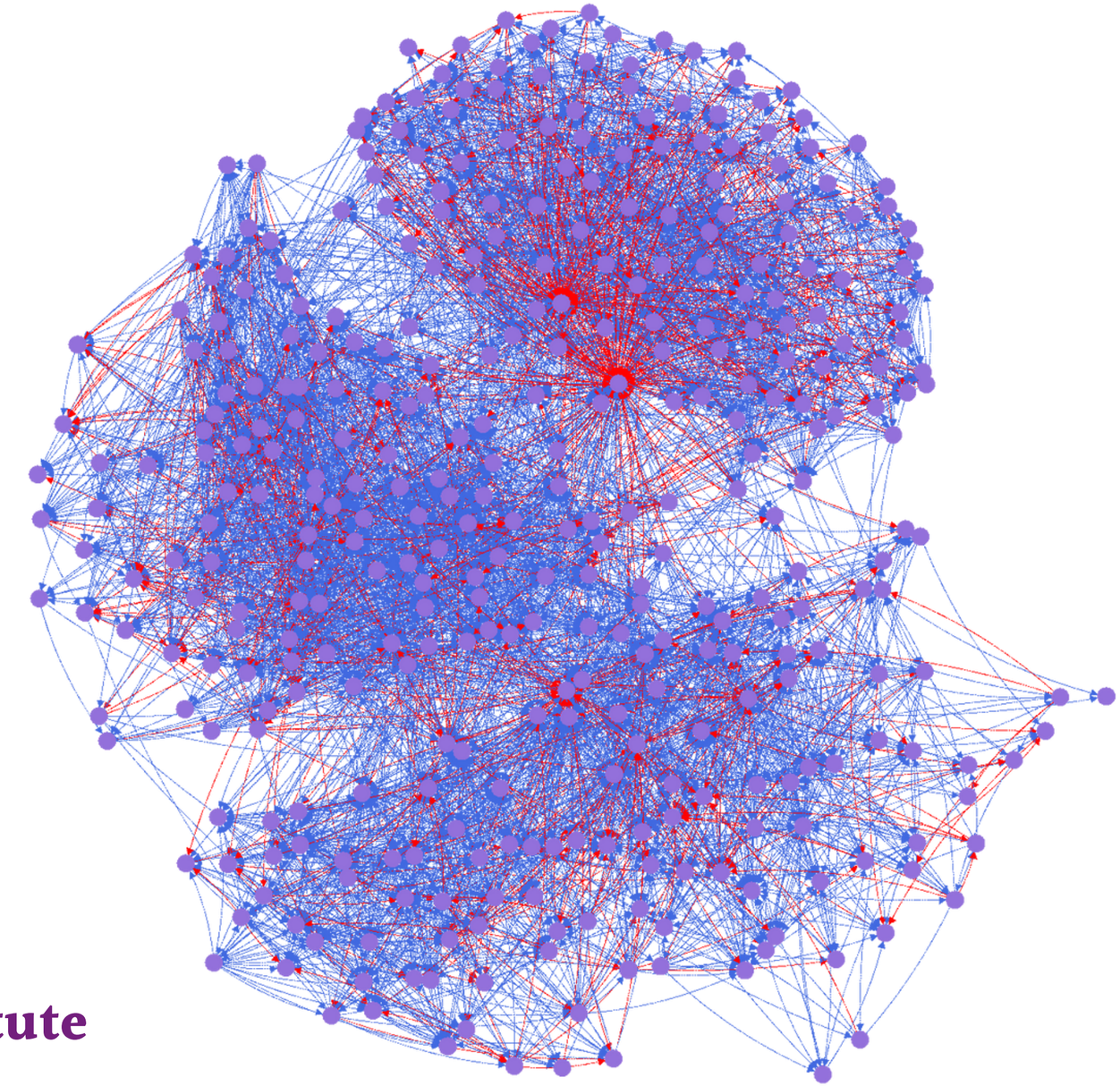
Agent-based model

ABM model that considers different games and phenotypes

Community detection and balance

Frustration-based method for partial balance and comparison measures

High school friendship network



16%

48%

1%

33%

2%

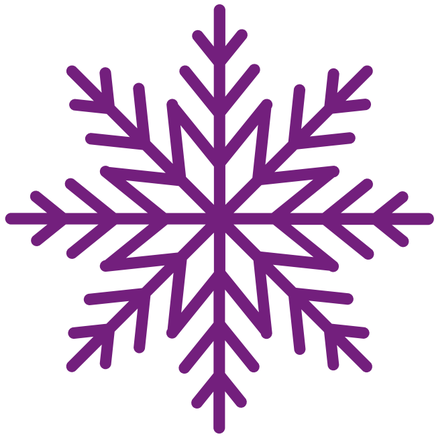
364 kids

Blas de Otero Institute

Social dilemmas

Prisoner's Dilemma

Stag Hunt Game



Snowdrift Game

Harmony Game

Poncela-Casasnovas, Julia, et al. "Humans display a reduced set of consistent behavioral phenotypes in dyadic games." Science advances 2.8 (2016): e1600451.

The model

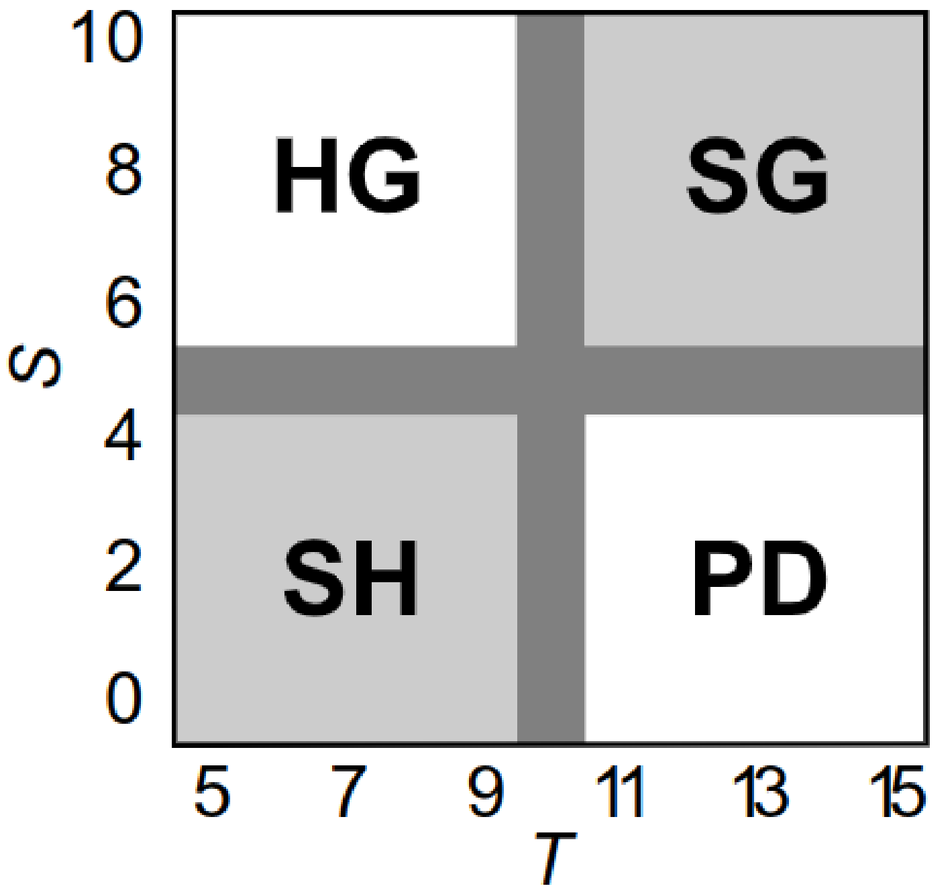
Payoff parameters:

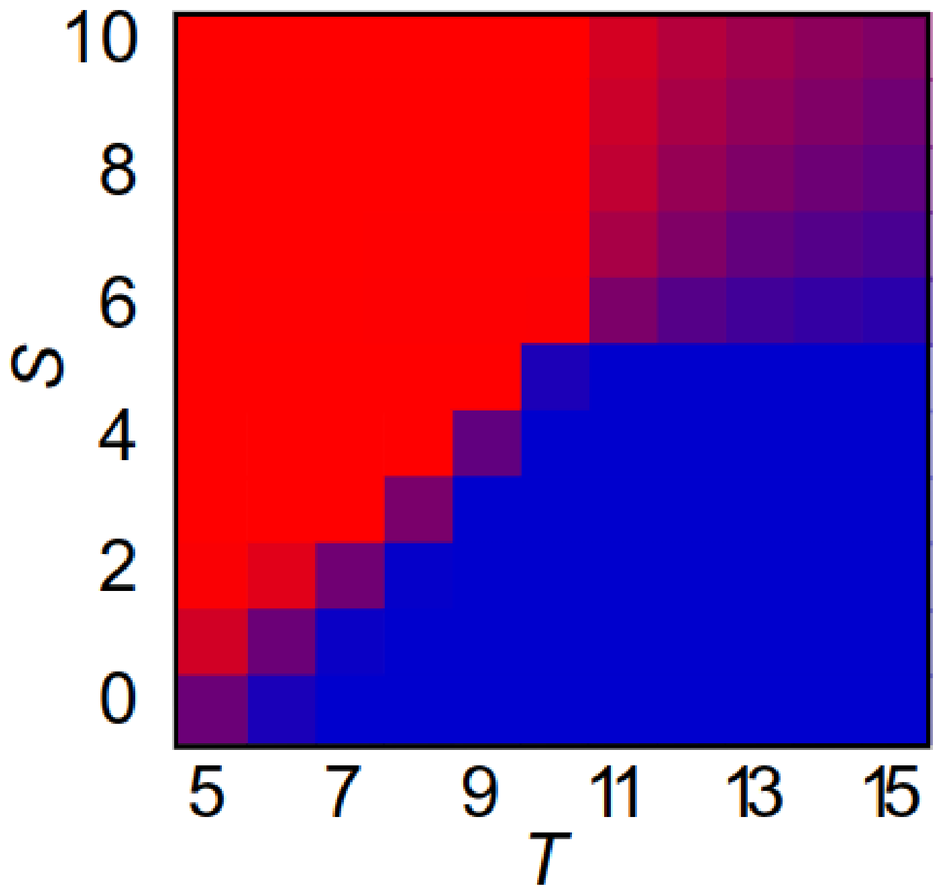
Reward: R = 10

Punishment: P = 5

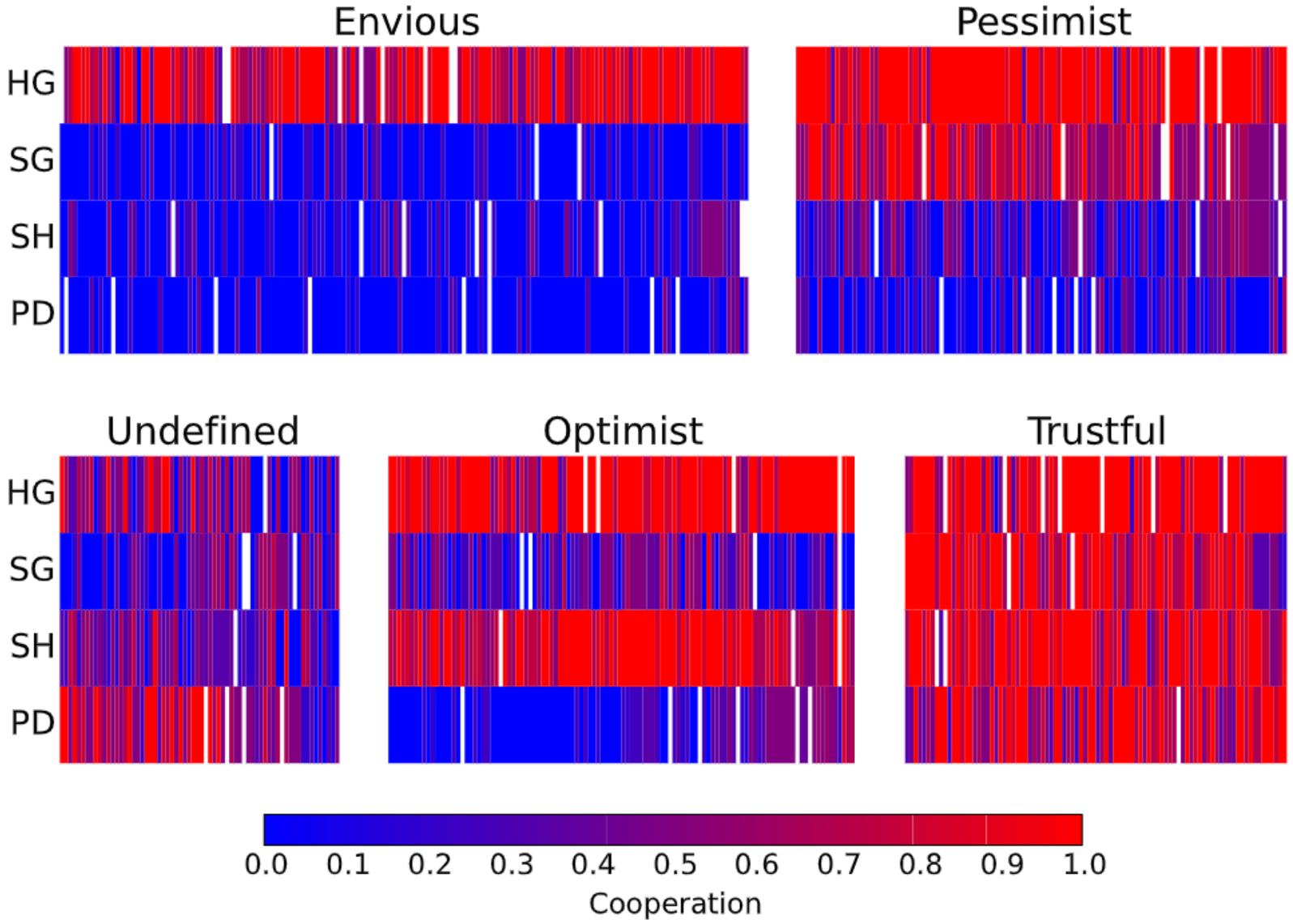
Temptation: 5<T<15

Sucker: 0<S<10





The myth of "rational" strategies



Phenotype proportions

Envious: 30%

Optimist: 20%

Undefined: 12%

Pessimist: 21%

Trustful: 17%

Behavioural phenotypes

Optimist

Maximizes max payoff

Pessimist

Maximizes min payoff

Envious

Defeats opponent

Trustful

Always cooperates

Undefined

Chooses randomly

Minimal model

Algorithm

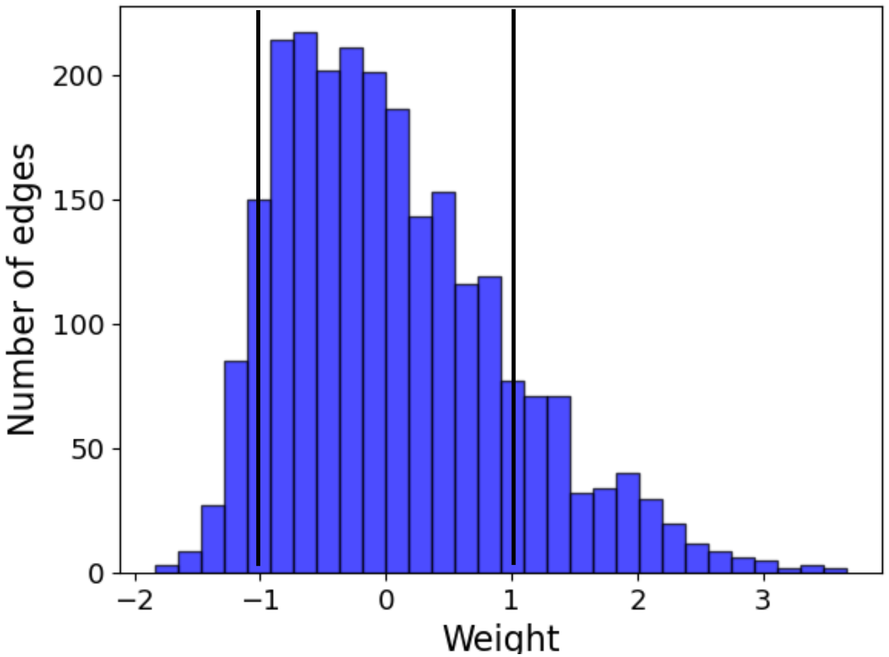
In each time step:

- Select *two nodes*, and payoffs **S** and **T**.
- Let the selected nodes *play*.
- Add *payoff* to the adjacency matrix element.

After the simulation:

- *Tipify* the adjacency matrix.
- Disconnect links *below* the thresholds.

Weight distribution for the minimal model

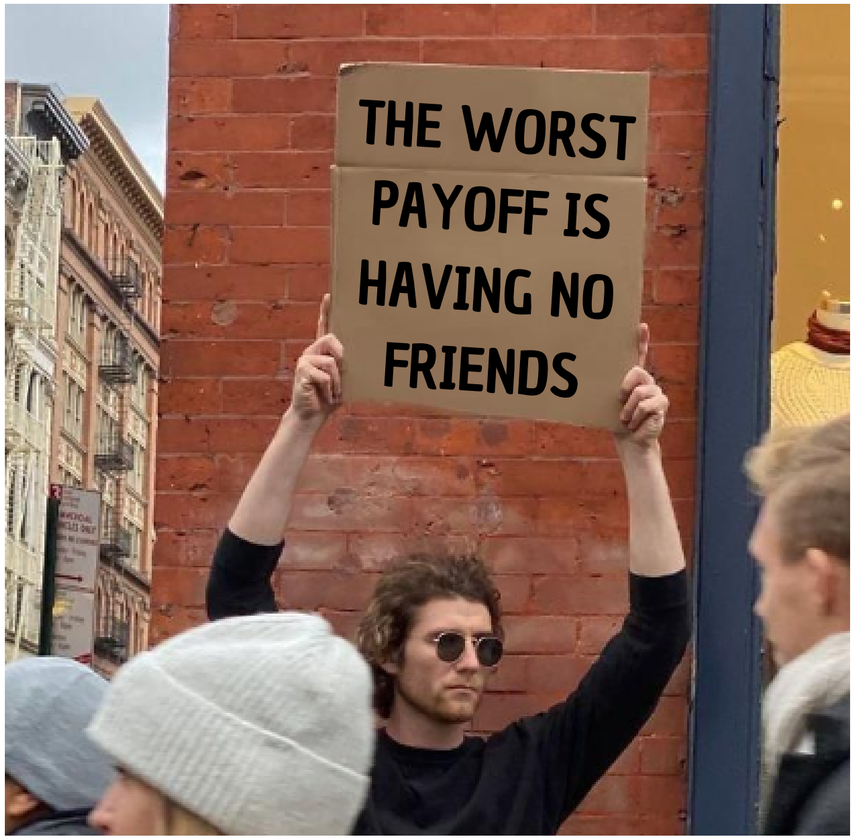


Edge removal model

Algorithm

In each time step:

- Select two nodes **i** and **j**, and payoffs **S** and **T**.
- Let the selected nodes play.
  - ***i - j connected:*** update edges (i,j) and (j,i) with the new payoff.
  - ***i - j not connected:*** create a link if the new payoff is larger than the average payoff of node i or if i has no friends.
- **Remove** edge with lowest weight with probability *k_i/kmax*.



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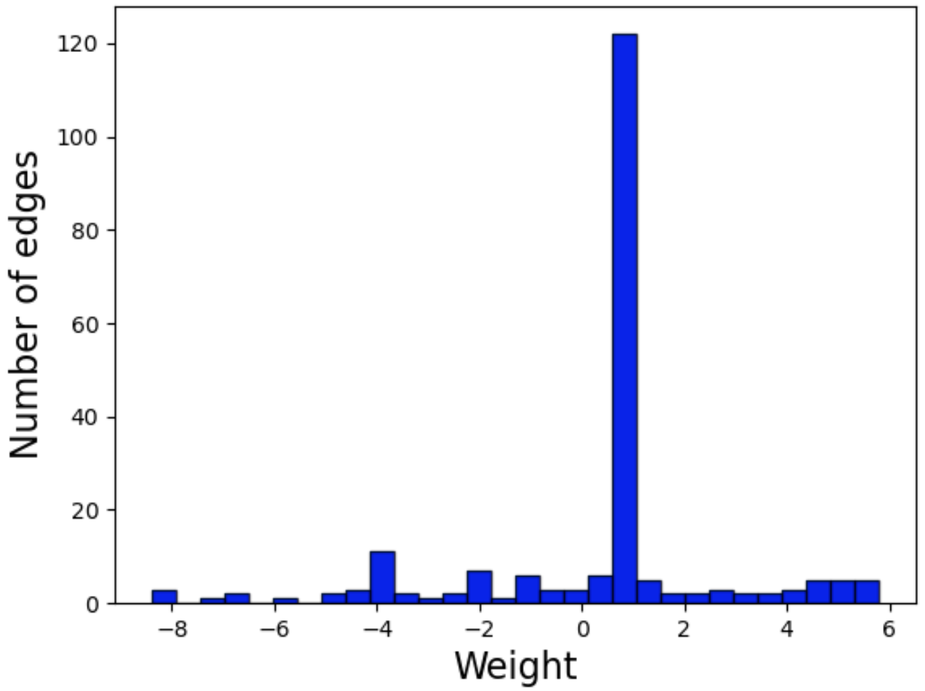
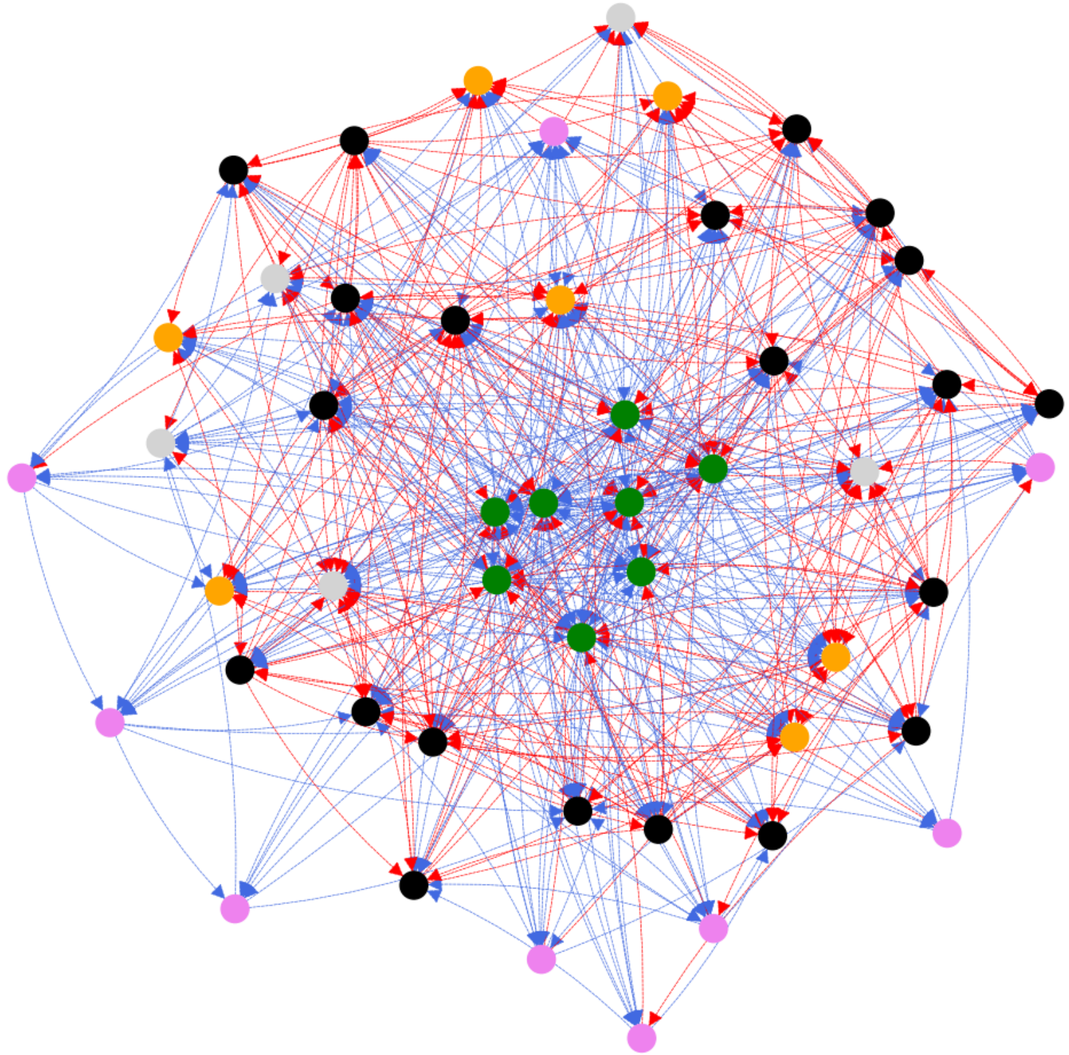
Weight distribution for the edge removal model



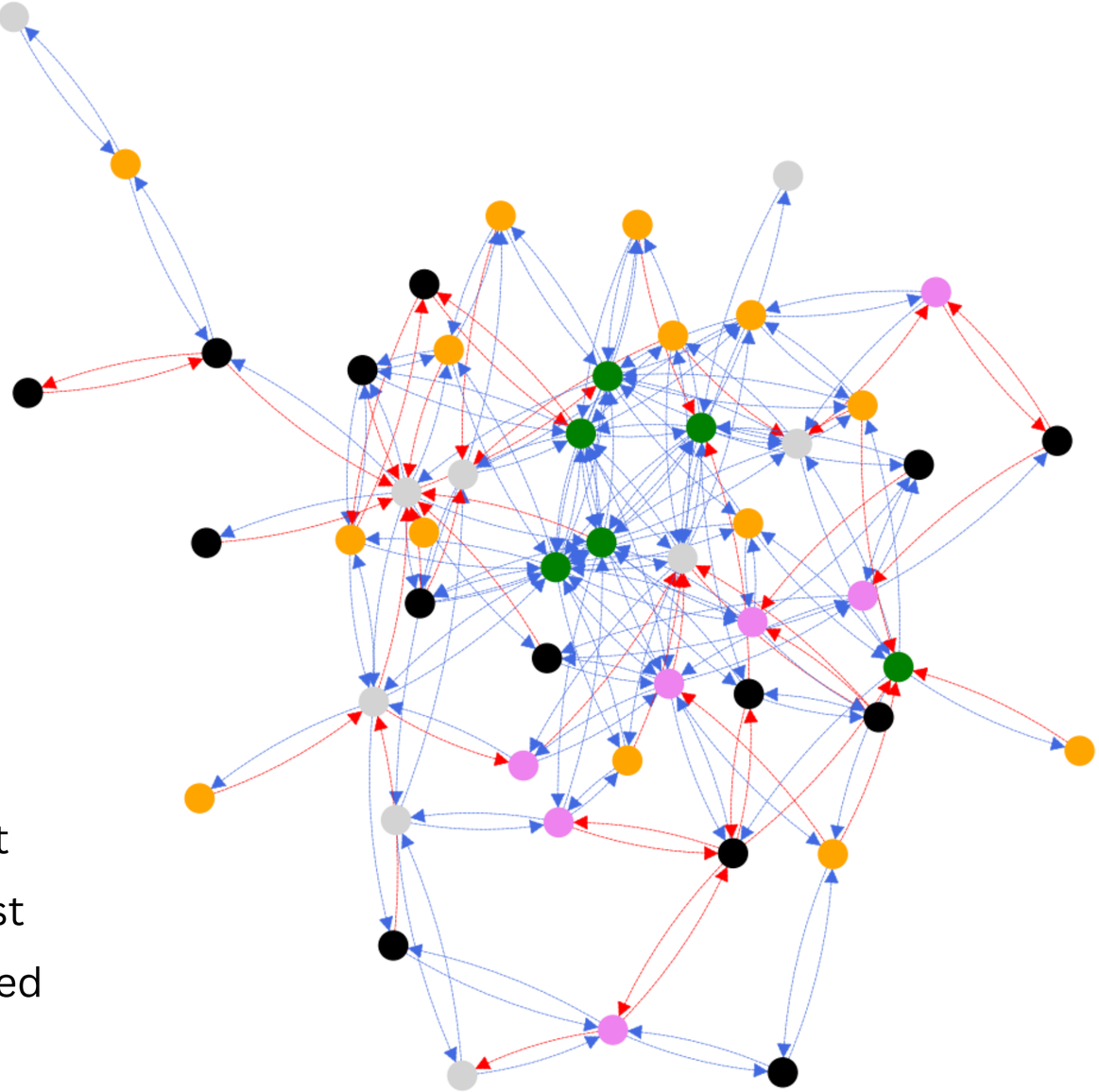
Results: minimal model

- **Trustful** nodes have *positive* in-degree yet *negative* out-degree.
- **Envious** nodes participate in most negative interactions.
- **Pessimist** nodes make friends easily!

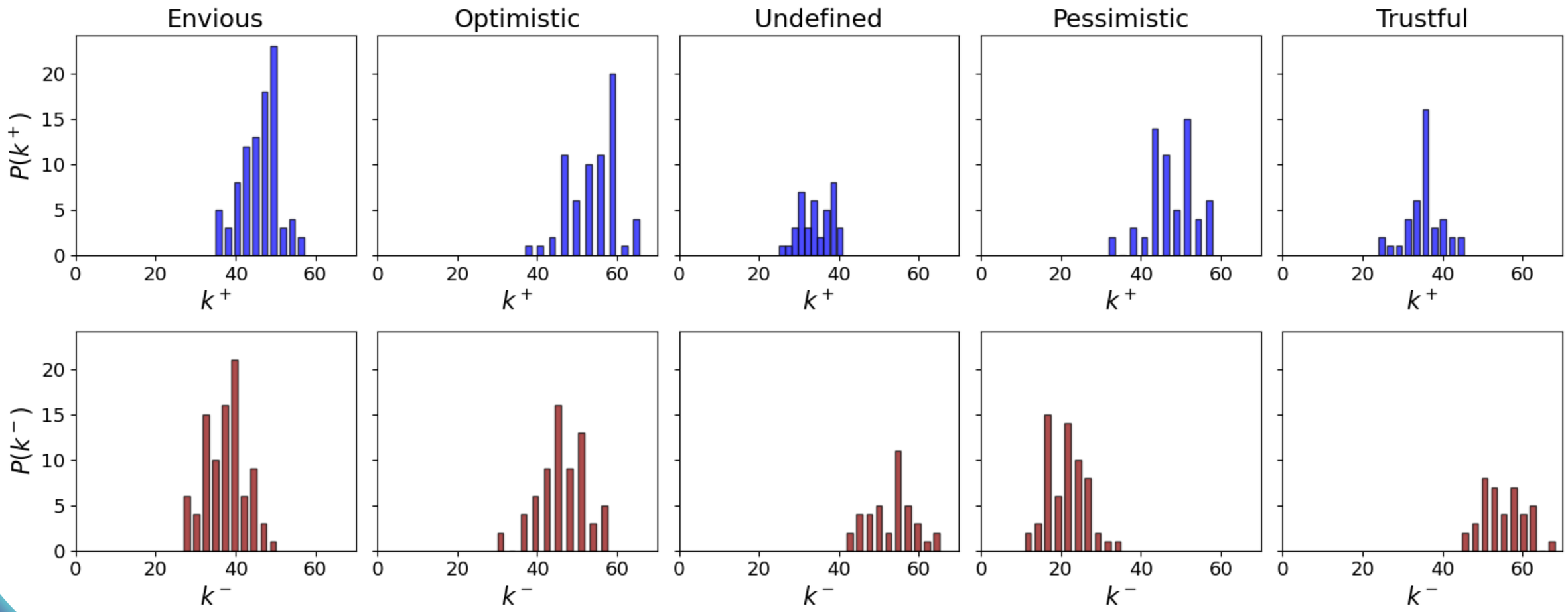


Results: link removal model

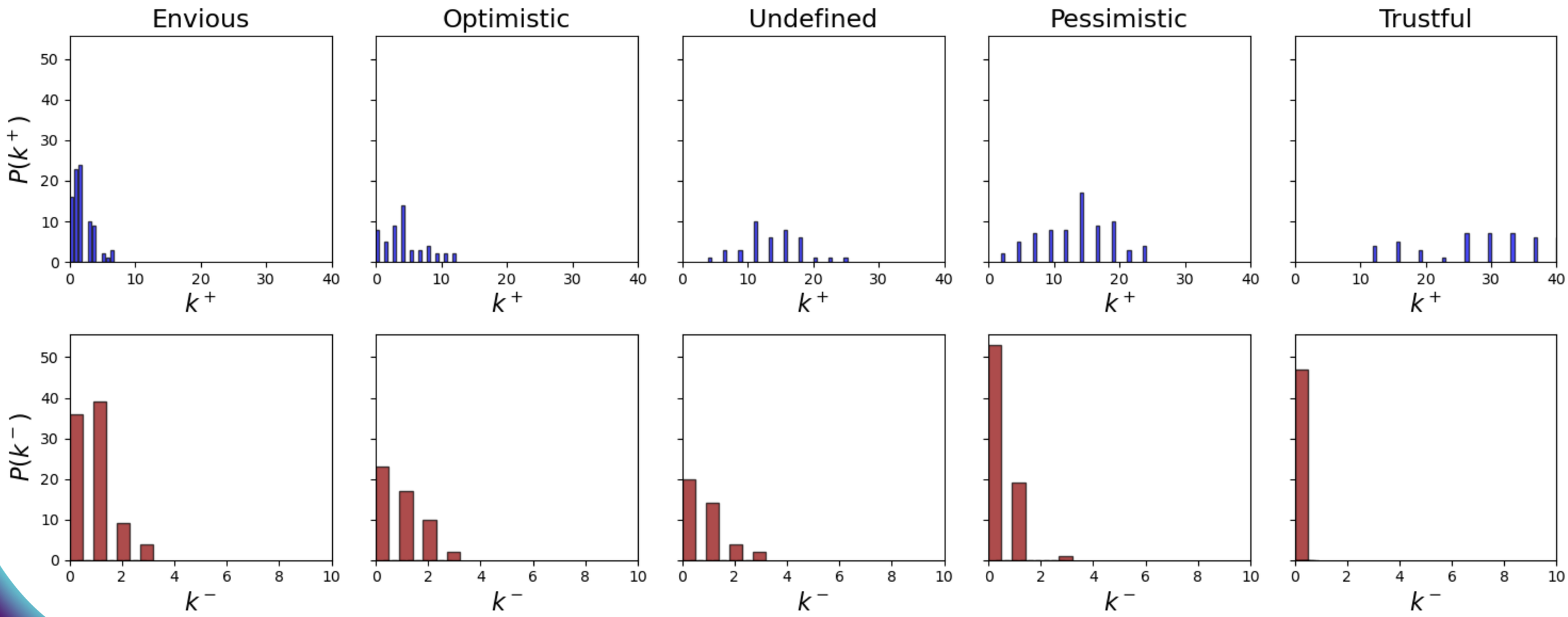
- Sparser network due to edge removal
- **Trustful** nodes are densely connected
- Edges are reciprocal but their signs can be antirreciprocal.



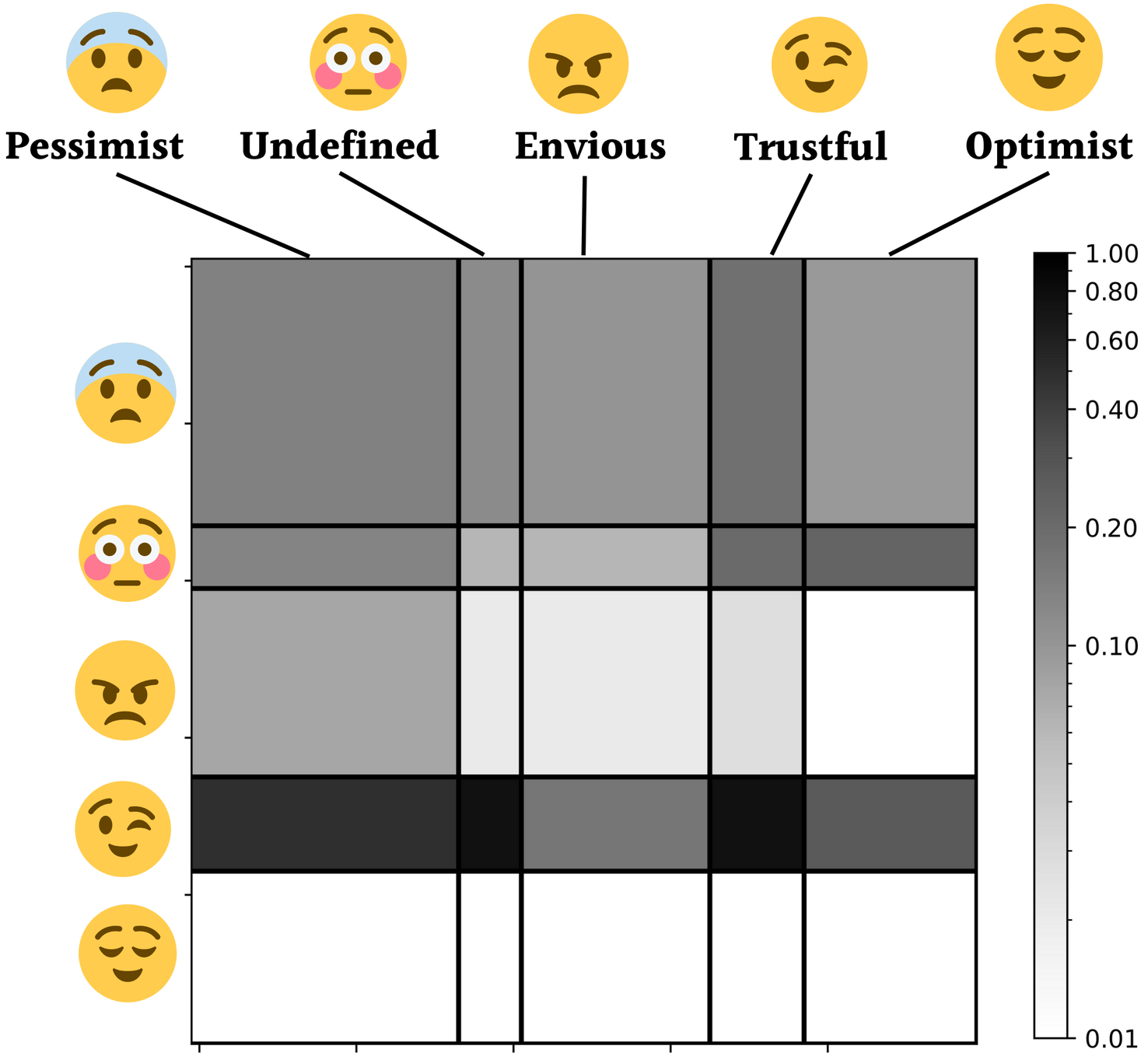
In-degree distributions (minimal model)



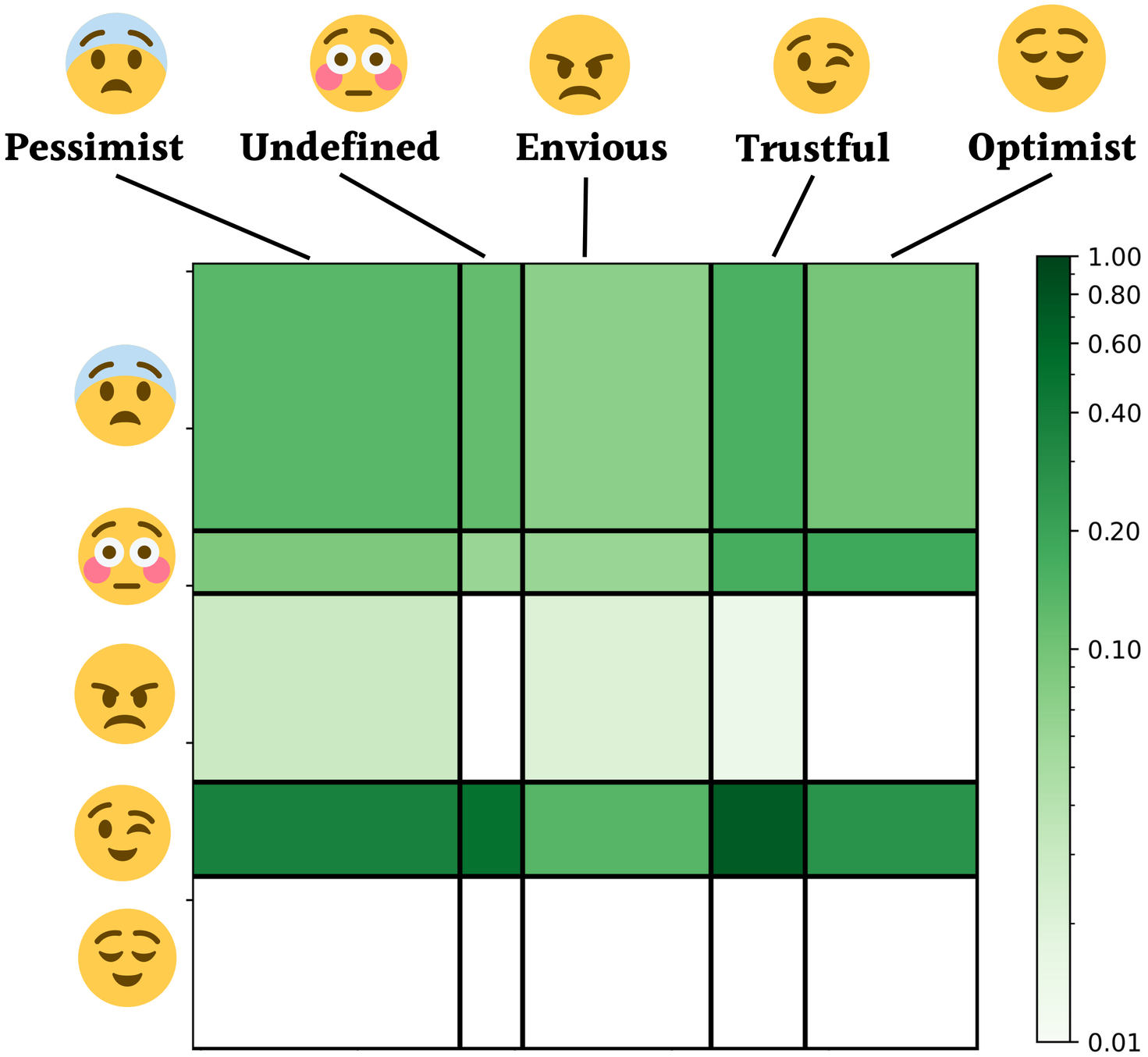
In-degree distributions (link removal)



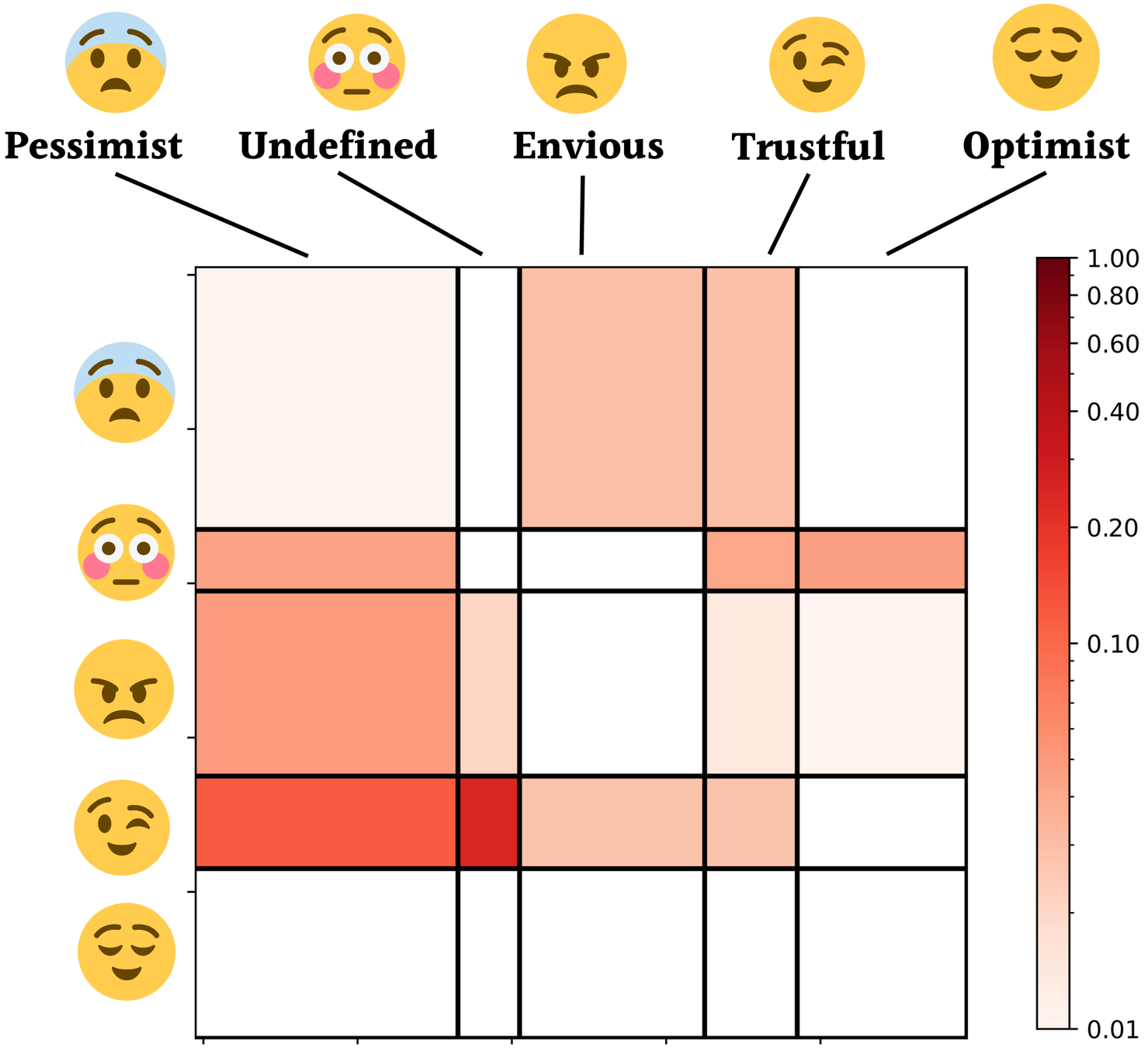
Total interaction among phenotypes



Positive interaction among phenotypes



Negative interaction among phenotypes



Comparison of community detection methods

Different methods:

- Stochastic block model with edge covariates
- 3-states Glauber Model
- Frustration based model
- Spinglass based model
- Spectral clustering

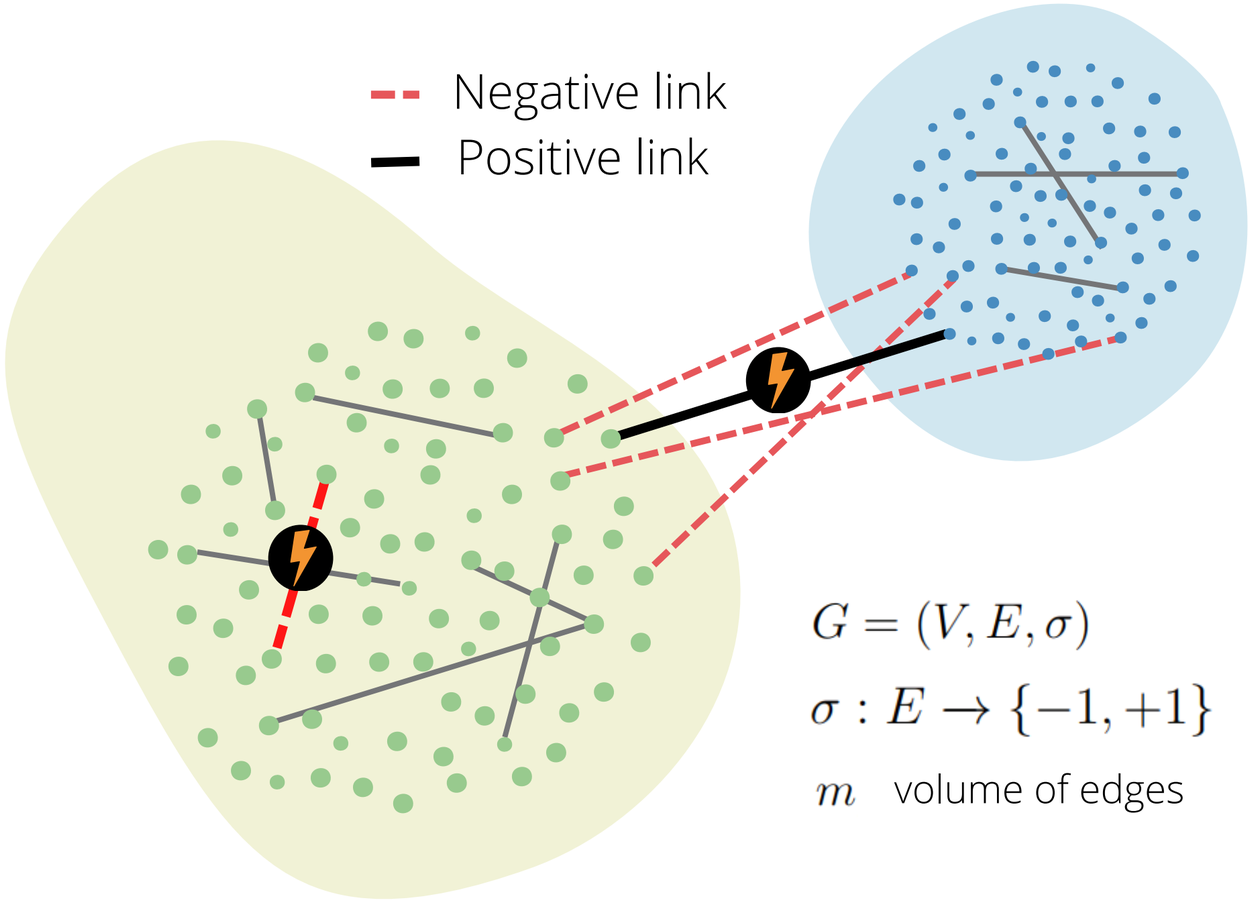
Similarity among methods:

- Rand Index
- Adjusted Rand Index
- Normalized mutual information

After choosing the two most similar methods,
compare the communities found by the two methods
using the Jaccard Index

Rahiminejad, S. et al. Topological and functional comparison of community detection algorithms in biological networks. BMC Bioinformatics 20, 212 (2019)

Frustration based methods for partial balance



$$G = (V, E, \sigma)$$

$$\sigma : E \to \{-1, +1\}$$

m volume of edges

Choice of k (number of groups)

Balance relates to the minimum number of frustrated edges

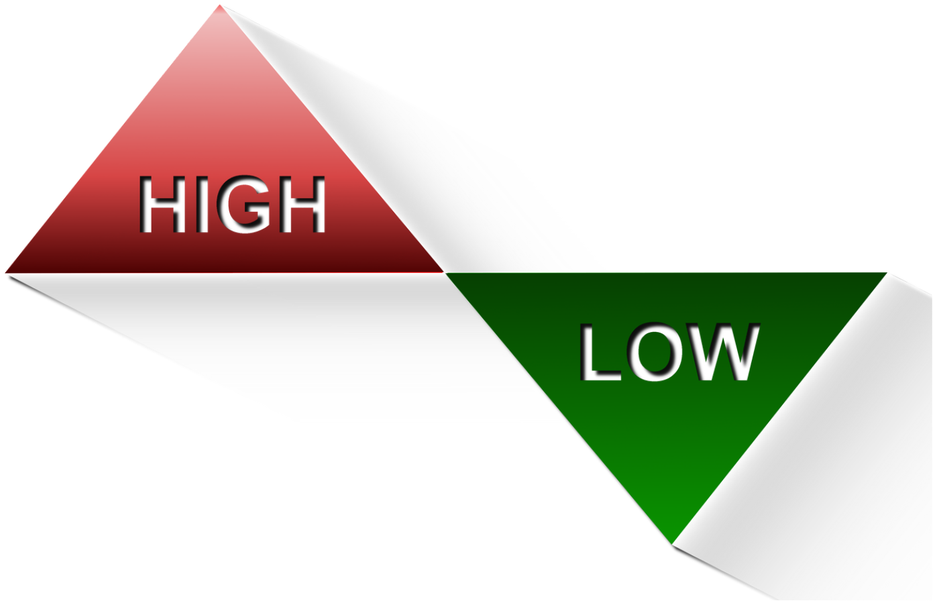
Balance measures

A.METHOD 1
Estrada - Benzi balance index

B.METHOD 2
S.Aref Clusterability index (frustration)

Synthetic network

0.68 (A) / 1 (B)



Real network

0.02 (A)

Take home messages:

- Friendship and enmities -> **game-theoretical payoffs** (social capital theory).
- Trustful players have the **largest in-degree**: cooperation is a winning strategy in coevolving networks.
- As a consequence, **cooperation** is the dominant strategy.
- The generative model produces **asymmetric relationships** between different phenotypes.
- Real friendship networks seem to have an extremely **low balance** in all measures.
- Real friendship networks have a community structure **independent of the balance**

Disclaimer: social capital theory can explain Shakira and Piqué break-up in terms of envious-optimist interactions.

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Time for questions!